Longitudinal Dynamics in Non-Scaling FFAGs

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FFAG03, KEK

Basic Equations

- Basic equations in time τ relative to the RF phase and energy E, independent variable is arc length s:

$$\frac{d\tau}{ds} = \frac{1}{L}[T(E) - T_0] \qquad\qquad \frac{dE}{ds} = v\, c(\omega\tau)$$

  - ω is angular RF frequency
  - v is RF gradient
  - $c(\phi)$ is RF voltage as a function of phase
  - L is ring circumference (cell length)
  - $T(E)$ is the time to make one turn (one cell), relative to RF phase

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Basic Equations (cont.)

Muon Collaboration

- (cont.)
  - T_0 is a time offset. It is easily adjusted to whatever you want
    - ⋆ Coarse adjustment: adjust relative cavity phases.
      - ➢ N equally spaced cavities, change by multiples of $2\pi/(N\omega)$.
      - ➢ $N = 50$, $\omega = 2\pi \cdot 200$ MHz, size of adjustment is 100 ps
    - ⋆ Fine adjustment: adjust cell length (if you're really picky)
      - ➢ Above numbers, change is at most 1.5 cm
- Change to dimensionless variables

$$x = \omega\tau \qquad\qquad p = \frac{E - E_{\text{min}}}{E_{\text{max}} - E_{\text{min}}}$$

  - x has range $-\pi$ to π, p has range 0 to 1

Scaled Equations

- Equations in dimensionless variables

$$\frac{dx}{ds} = \lambda(p) - \lambda_0 \qquad\qquad \frac{dp}{ds} = \frac{v}{\Delta E} c(x)$$

  - $\Delta E = E_{\text{max}} - E_{\text{min}}$
  - $\lambda(p) = \omega T(E_{\text{min}} + p\Delta E)/L$: dimensions of inverse length
  - $\lambda_0 = \omega T_0/L$: dimensions of inverse length

- Parabolic time-of-flight variation, with minimum at central energy:

$$\lambda(p) = \frac{\omega \Delta T}{L}(2p-1)^2$$

  - Maximum value on $[0,1]$ is $\omega\Delta T/L$

Scaled Equations

- Change independent variable to $u = \omega \Delta T s / L$

$$\frac{dx}{du} = (2p - 1)^2 - z \qquad \frac{dp}{du} = wc(x)$$

- Equations depend on two dimensionless parameters

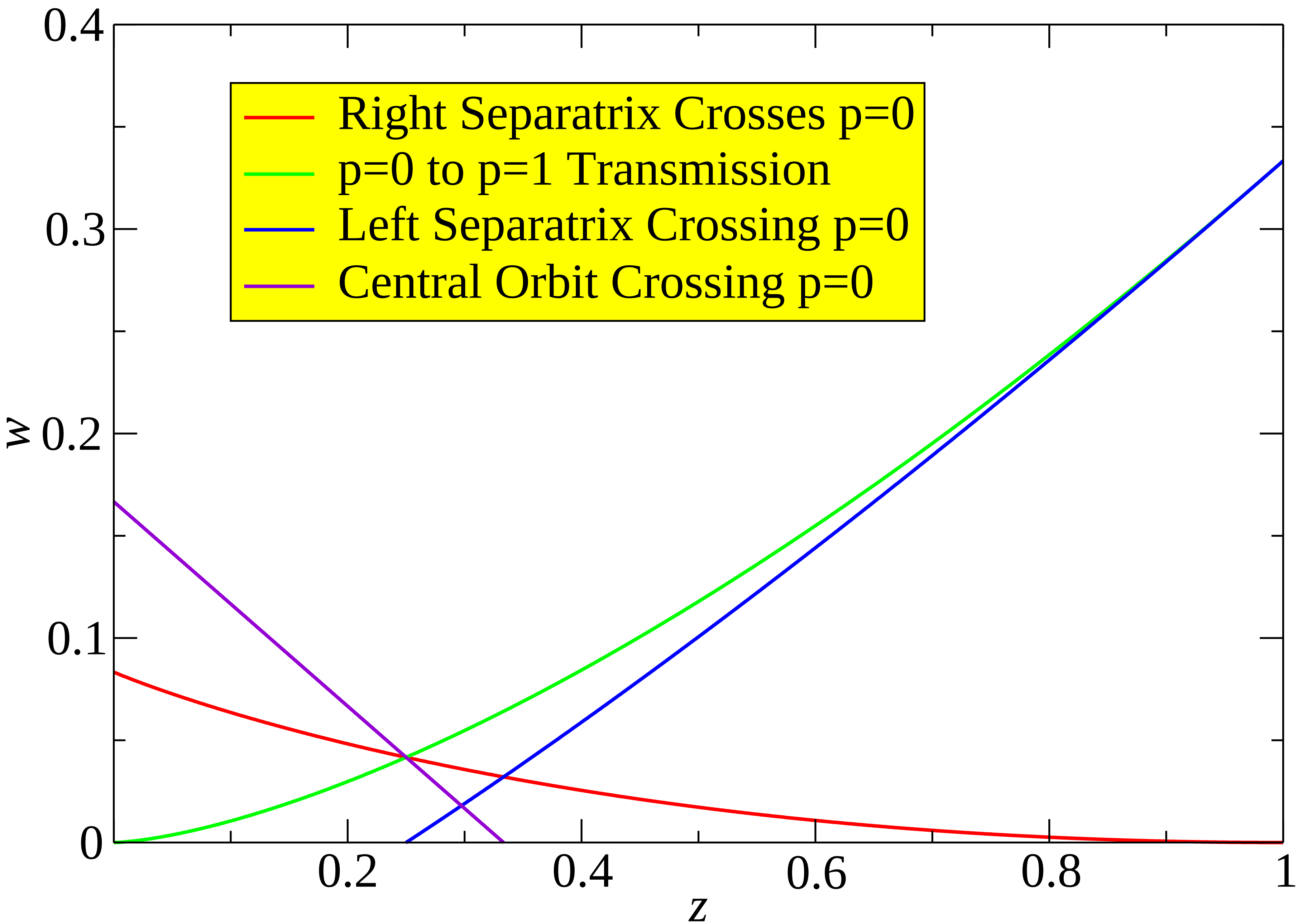
$$z = \frac{T_0}{\Delta T} \qquad w = \frac{vL}{\omega \Delta T \Delta E} = \frac{V}{\omega \Delta T \Delta E}$$

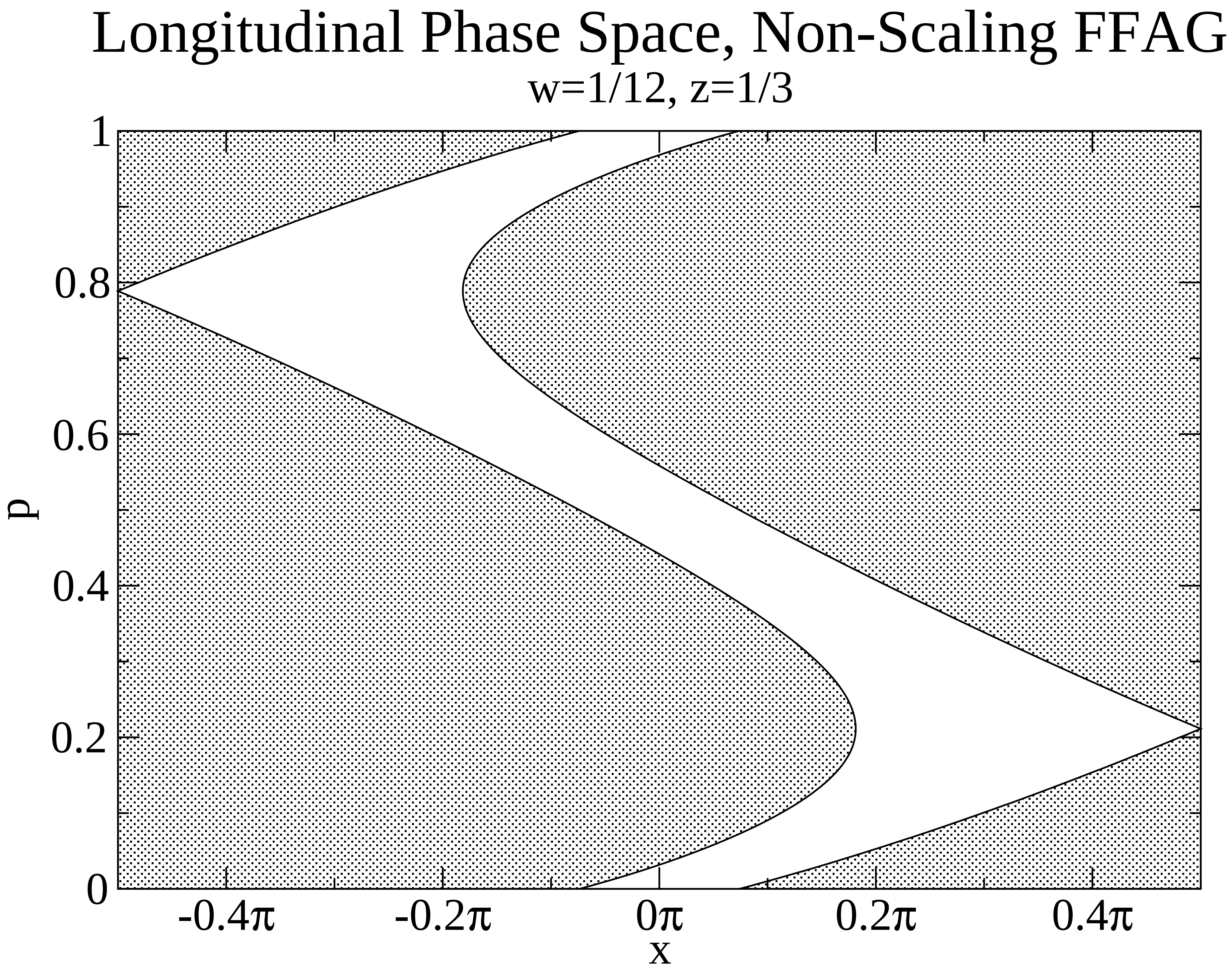
  - V is total voltage in same length that ΔT is maximum time of flight
  - Multiply distances in u by $L/\omega\Delta T$

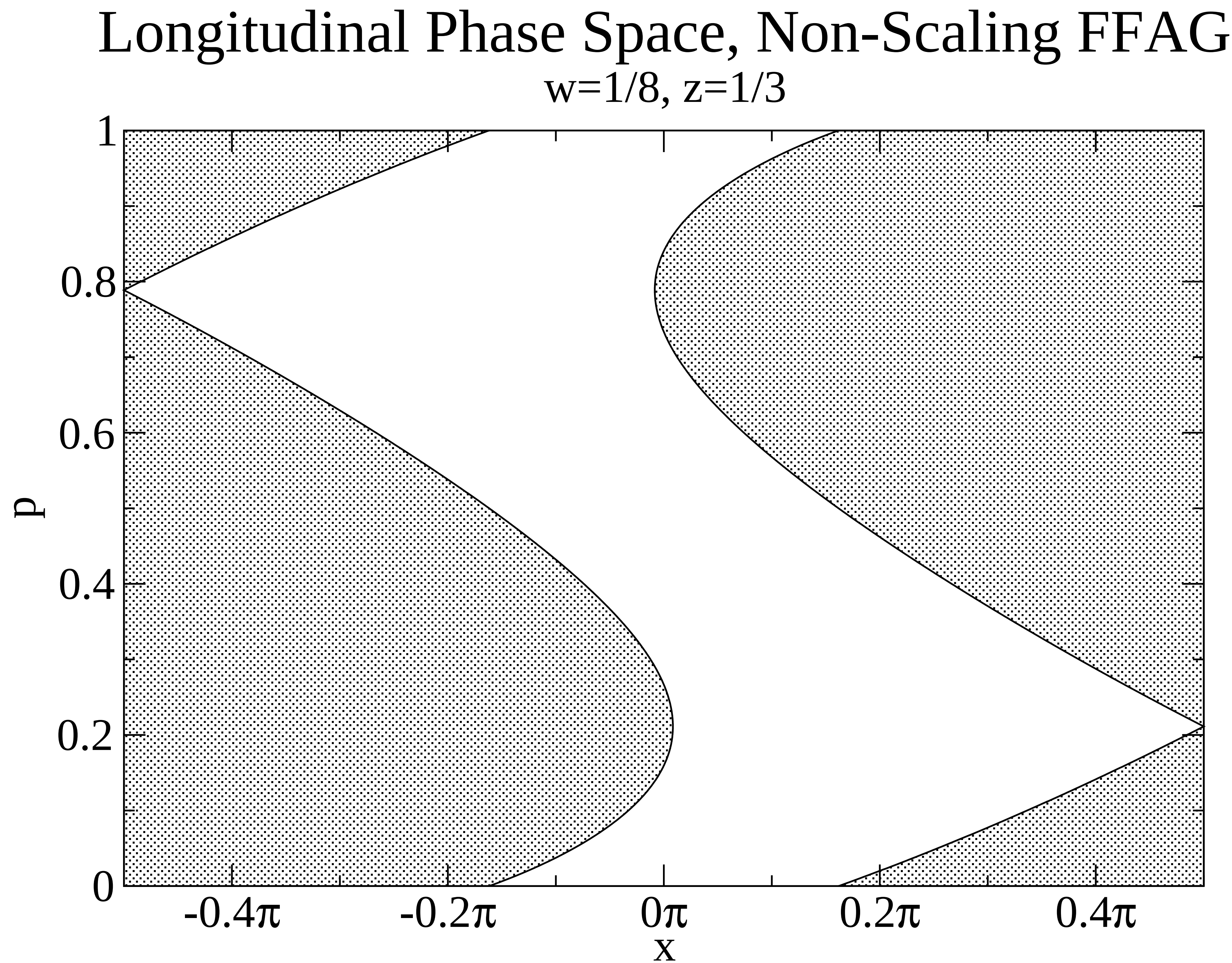
Parameter Space

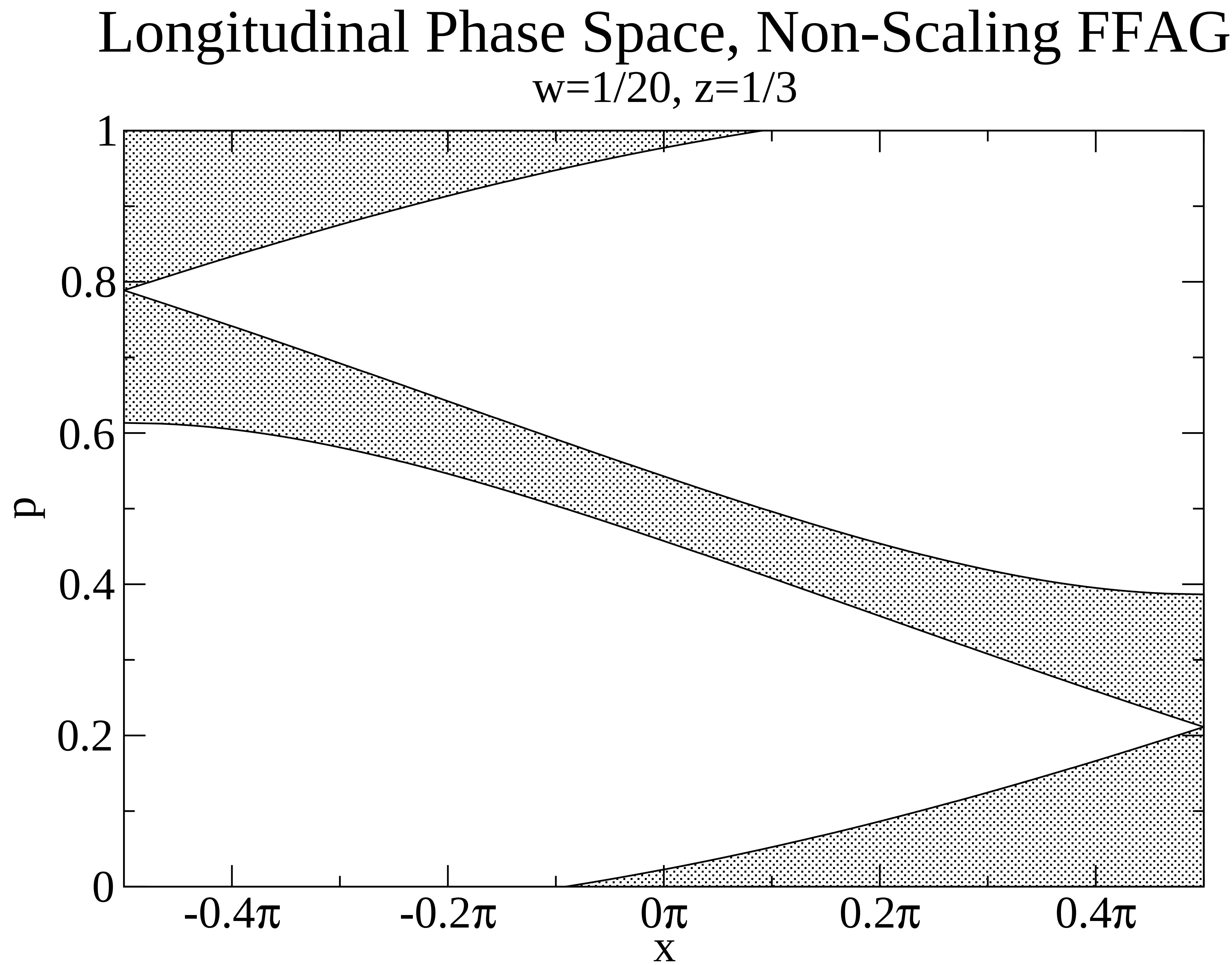
- There are two separatrices. At least one must cross the $p = 0$ axis to capture particles.
- If the separatrices are not oriented properly, particles will not be able to get from $p = 0$ to $p = 1$.
- Probably don't want situation where central orbit does not cross $p = 0$ axis: symmetry
- Need to study optimum phase space transmission in this parameter space
  - Best z for given w
    - $z = 1/4$ seems optimal, but not sure
      - Largest distance from minimum at fixed z
      - Central orbit: maximum phase swing same as initial phase
  - Optimal injection distribution

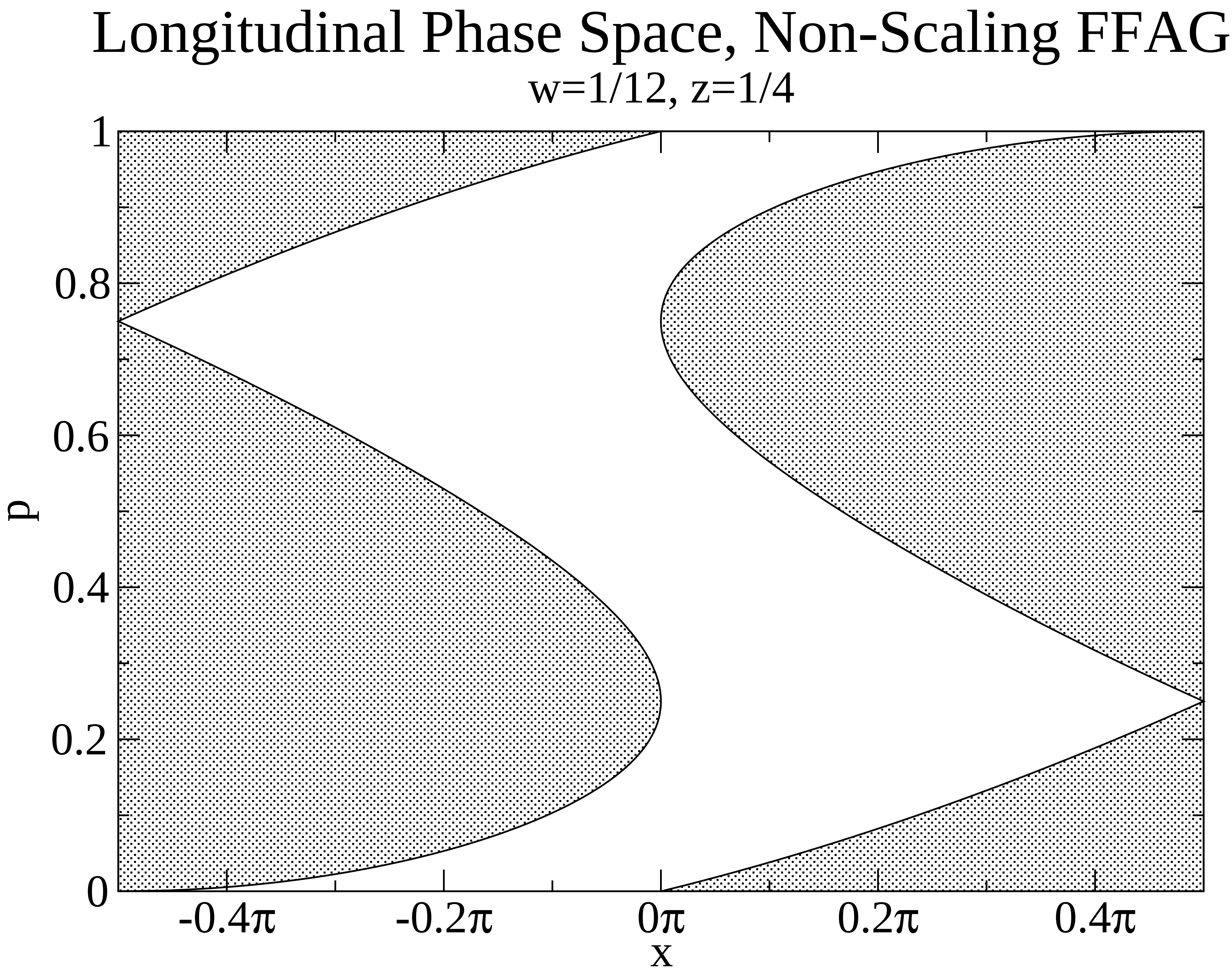
Parameter Space for Parabolic Time-of-Flight

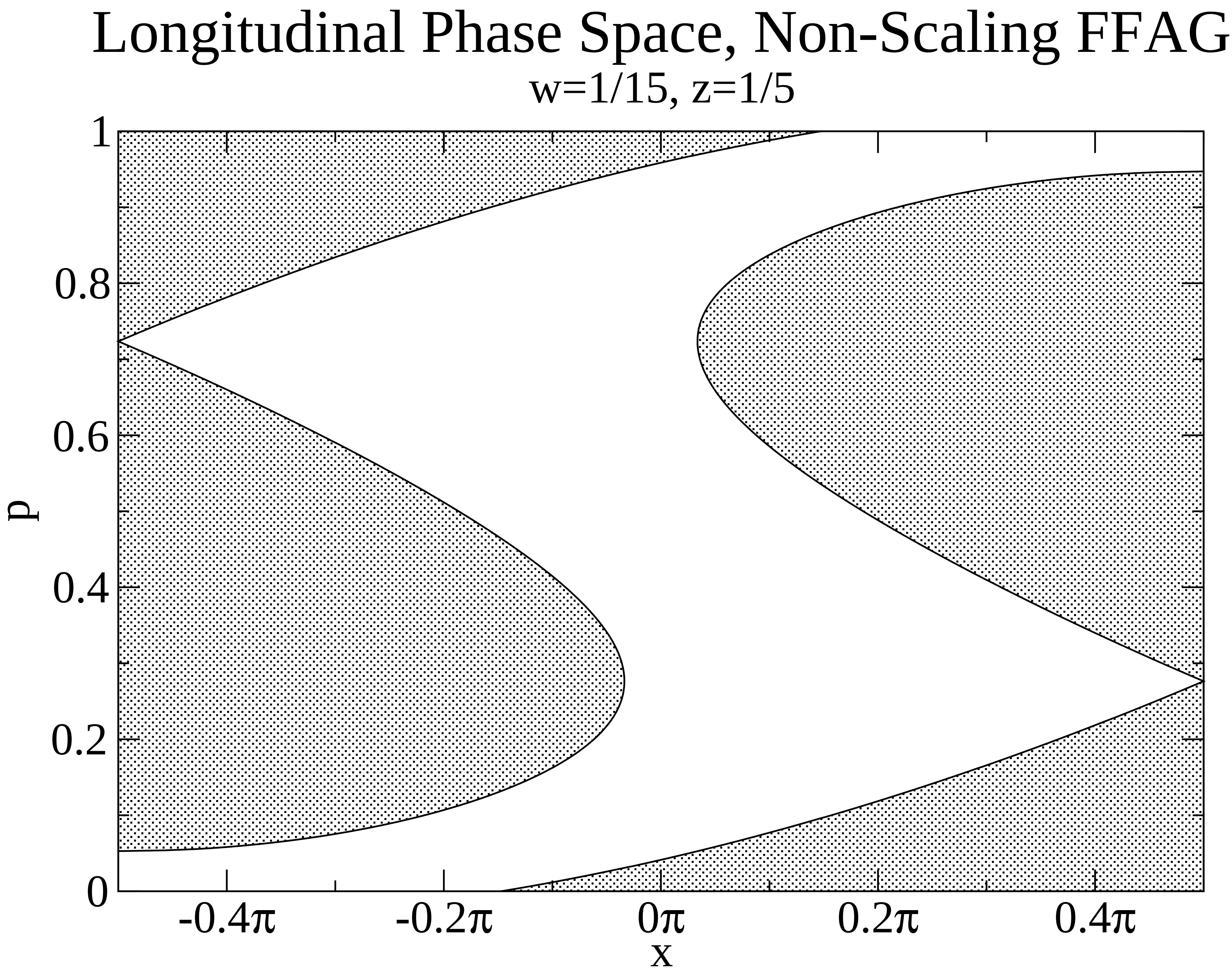



Longitudinal Phase Space, Non-Scaling FFAG
w=1/12, z=1/3
p
x
1
0.8
0.6
0.4
0.2
0
-0.4π
-0.2π
0π
0.2π
0.4π


Longitudinal Phase Space, Non-Scaling FFAG
w=1/8, z=1/3
p
0
0.2
0.4
0.6
0.8
1
-0.4π
-0.2π
0π
0.2π
0.4π
x


Longitudinal Phase Space, Non-Scaling FFAG
w=1/20, z=1/3
1
0.8
0.6
0.4
0.2
0
p
-0.4π
-0.2π
0π
0.2π
0.4π
x

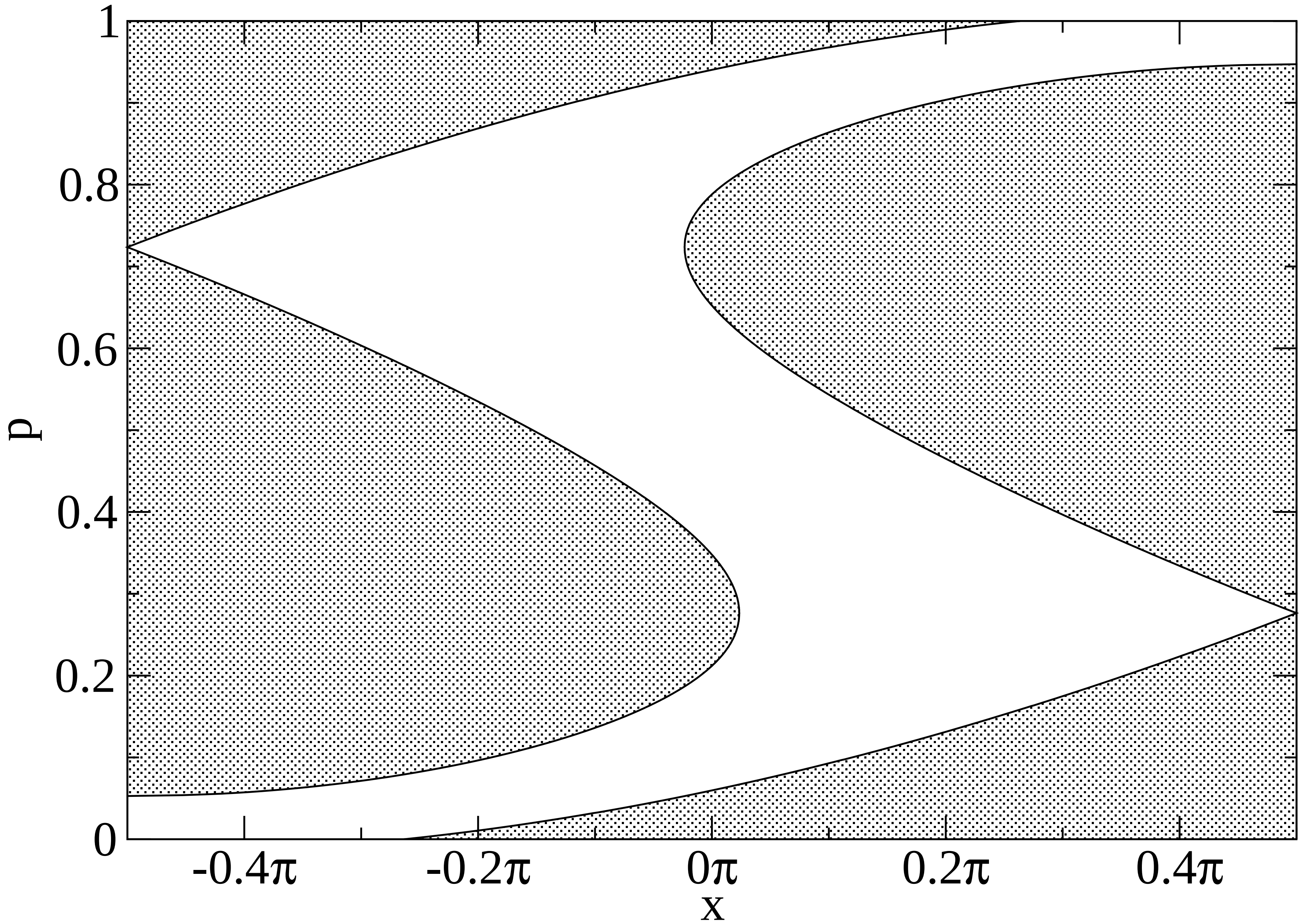

Longitudinal Phase Space, Non-Scaling FFAG
w=1/12, z=1/4
1
0.8
0.6
0.4
0.2
0
p
-0.4π
-0.2π
0π
0.2π
0.4π
x



Longitudinal Phase Space, Non-Scaling FFAG
w=1/15, z=1/5
1
0.8
0.6
0.4
0.2
0
p
-0.4π
-0.2π
0π
0.2π
0.4π
x

Longitudinal Phase Space, Non-Scaling FFAG

w=1/18, z=1/5



Longitudinal Phase Space, Non-Scaling FFAG

w=1/12, z=1/4